Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 19, 2003*
Accepted: *January 16, 2004*

Reissner-Nordström-de Sitter black hole, planar coordinates and dS/CFT

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Abstract: We discuss the Reissner-Nordström-de Sitter black holes in the context of dS/CFT correspondence by using static and planar coordinates. The boundary stress tensor and the mass of the solutions are computed. Also, we investigate how the RG flow is changed for different foliations. The Kastor-Traschen multi-black hole solution is considered as well as AdS counterparts of these configurations. In particular, we find that in planar coordinates the black holes appear like punctures in the dual boundary theory.

Keywords: AdS-CFT and dS-CFT Correspondence, Black Holes.

http://jhep.sissa.it/archive/papers/jhep012004029/jhep012004029.pdf

Contents

1. Introduction

There are a number of observational as well as theoretical reasons that motivate, at least partially, the recent interest in asymptotically de Sitter (dS) spacetimes. The observational evidence accumulated in recent years (see, e.g., ref. [1]), seems to favour the idea that the physical universe has an accelerated expansion. The most common explanation is that the expansion is driven by a small positive vacuum energy (cosmological constant, $\Lambda(G\hbar/c^3) \approx 10^{-123}$), implying the spacetime is asymptotically dS. Furthermore dS spacetime plays a central role in the theory of inflation (the very rapid accelerated expansion in the early universe), which is supposed to solve the cosmological flatness and horizon puzzles.

dS spacetime is the maximally symmetric lorentzian space (i.e. the number of Killing vector fields is the same as for flat spacetime) with constant positive curvature. In D-dimensions the symmetry group is $\mathrm{SO}(1, D)$ and the topology is $\mathbb{R} \times S^{D-1}$. The existence of compact Cauchy surfaces (spatial slices), easier seen from the topology of dS spacetime, makes unclear the procedure to define conserved charges. The causal structure of dS spacetime is such that inertial observers are surrounded by cosmological horizons. The expansion is so rapid that no signal originating outside any observer's horizon will ever reach her; equivalently there is a null surface that cannot be crossed by any material particle or signal. If an observer does not have access to a part of the spacetime, then she will attribute an entropy to the gravitational field because of the microscopic degrees of freedom that are hidden.

A cosmological horizon is a characteristic of many spacetimes having a positive cosmological constant. The next logical step is to define an energy for such a spacetime in order to provide consistent thermodynamic relationships, a problematic task in general relativity.

One of the most fruitful approaches in computing conserved quantities has been to employ the quasilocal formalism [2]. The basic idea here is to enclose a given region of spacetime with some surface, and to compute all relevant (conserved and/or thermodynamic) quantities with respect to that surface [3, 4]. For a spacetime that is either asymptotically flat or asymptotically anti-de Sitter it is possible to extend the quasilocal surface to spatial infinity without difficulty, provided one incorporates appropriate boundary terms in the action to remove divergences [3]–[8]. For pure and asymptotically de Sitter spacetimes inside the cosmological horizon (where the Killing vector is timelike) computations of conserved charges and actions/entropies have been carried out [9].

The microphysical statistical origin of cosmological horizon entropy may well be associated with a holographic dual theory. Inspired by the celebrated anti-de Sitter (AdS)-conformal field theory (CFT) correspondence [10], some authors have conjectured a holographic duality between quantum gravity in an asymptotically dS spacetime and a CFT living on its boundary [11]–[15]. However, in spite of considerable recent activity, the precise nature of the proposed dS/CFT correspondence remains vague, with numerous conceptual issues remaining to be addressed. First, there is no satisfactory realization of dS space in string theory. Also, unlike in AdS, in asymptotically dS spacetimes there is no asymptotic Killing vector that is globally timelike. Consequently the hamiltonian is not positive definite and there is no spatial infinity — the asymptotic regions are euclidean surfaces (spheres) at past and future temporal infinities [16]. The conjectured dS/CFT correspondence introduces holographic screens at timelike past infinity $\mathcal{I}^-$ and/or timelike future infinity $\mathcal{I}^+$ (see, e.g., ref. [17] for a different approach — the holographic screens, at the cosmological horizons, are observer dependent). The theory on the screen is necessarily a euclidean CFT, with a scale that encodes the dimension transverse to the screen.

Despite these complications, we have learned a lot about asymptotically dS spacetimes in recent years. A novel method for renormalizing the stress-energy of gravity that provides a measure of the gravitational mass (and the boundary stress tensor) was proposed by Balasubramanian, de Boer and Minic in ref. [18] (referred to thereafter as the BBM prescription). This method is analogous to the Brown-York prescription in asymptotically AdS spacetimes: one supplements the quasilocal formalism by including boundary counterterms that depend on the curvature invariants. By generalizing the Gibbs-Duhem relation, a definition of entropy outside the cosmological horizon was formulated in ref. [19]. This formalism was recently used to study the thermodynamic properties of four-dimensional Kerr-Newman-dS black holes in stationary coordinates [20].

Because of its high degree of symmetry, dS space has a simple form in a large number of coordinate systems. There is a static frame centered on each observer (timelike geodesic) in dS. When a black hole exists, there is still a static frame centered about the black hole. Since different parametrizations emphasize different features, it is of interest to study physics in dS space in alternative coordinate systems. For example, a multi-black hole solution in dS can be formulated in cosmological coordinates — one cannot build a static frame for this situation. Moreover, by choosing different foliations of the spacetime one can describe boundaries that have different topologies and geometries (metrics), affording study of the CFT on different backgrounds. Specifically, we find additional Casimir-type contributions to the total energy depending on the slicing topology in accord with the expectations from quantum field theory in curved space [21].

In what follows we will use two parametrizations of dS_{n+2} (with $n > 1$). The first we will refer to as static coordinates

$$ds^2 = (1 - H^2R^2)^{-1}dR^2 + R^2d\Omega_n^2 - (1 - H^2R^2)dT^2\,,$$

with $d\Omega_n^2 = \omega_{ab}dx^a dx^b = d\theta^2 + \sin^2\theta d\Omega_{n-1}^2$ the unit metric on S^n. Throughout the paper we set $c = G = 1$; also, the indices $\{a, b, \dots\}$ will indicate the angular coordinates and $\{i, j, \dots\}$ will indicate the intrinsic coordinates of the boundary metric. The cosmological constant is $\Lambda = n(n+1)H^2/2$. This parametrization is particularly useful, since the metric looks very simple and is time independent. The existence of a cosmological horizon at $R = 1/H$, with all thermodynamical properties, is here manifest [22].

The second set, which we will refer to as planar coordinates (i.e. spatial slices are flat) — or cosmological or inflationary coordinates — is given by

$$ds^2 = e^{2Ht}(dr^2 + r^2d\Omega_n^2) - dt^2\,.$$

In these isotropic coordinates, the expansion of the universe is explicit, and we can address questions relevant for a cosmology with a period of inflation. The relationship between these coordinate patches and their Penrose diagrams are presented in ref. [16].

Similar to the AdS case, black holes in dS space are expected to provide crucial information in understanding the dS/CFT correspondence. Most of the studies on cosmological black hole spacetimes make use of the static coordinates. It is less well-known that (electrovacuum-) black hole physics can also be discussed using a planar coordinate system. Although such metrics have explicit time-dependence, they also have a Killing vector that is equivalent to the $\partial/\partial T$ in static coordinates. Alternatively, the "static" metric is time-dependent outside of the cosmological horizon. Either way — in planar or static coordinates — it is a time dependent spacetime since there is no global timelike isometry.

A discussion of the relationship between planar and static coordinates for a Schwarzschild-de Sitter black hole, from the viewpoint of the dS/CFT correspondence conjecture, can be found in refs. [23]–[25]. The simplest generalization of the Schwarzschild black hole is to include a U(1) field in the theory. For a negative Λ, the corresponding Einstein-Maxwell (EM) solutions have proven useful in understanding various aspects of AdS/CFT [26]. The relevance of Reissner-Nordström-de Sitter (RNdS) black holes within the dS/CFT correspondence has been discussed by a number of authors, using a static coordinate system. However, the RNdS solution also admits a simple form in a planar coordinate system [27], which allows for a multi-black hole generalization given by Kastor and Traschen in ref. [28] (referred to as the KT solution). This solution describes an arbitrary number of charged black holes which, due to the presence of a cosmological constant, are in motion with respect to one another. In the following we re-examine these solutions in the context of the dS/CFT correspondence. Using the planar coordinate system, we find that the black holes appear as punctures in the dual theory, corresponding to divergences in the stress tensor of the CFT.

JHEP01(2004)029

The remainder of our paper is organized as follows: we start by reviewing in section 2 the counterterm formalism for asymptotically dS spacetimes. In subsection 2.1 we discuss the RNdS solution in static coordinates, compute its entropy working in a grand canonical ensemble and comment on its relationship to the Cardy-Verlinde formula of the *hypothetical* dual CFT. In subsection 2.2 we investigate the RNdS black hole in planar coordinates and in section 3 we describe the multi-black hole generalization in the context of the dS/CFT correspondence. A discussion of RG flow and the c-function for the RNdS solution is given in section 4. Using a double analytical continuation, we build in section 5 new time-dependent backgrounds with a negative cosmological constant. We conclude in section 6 with a discussion of our results.

2. Counterterm method and conserved charges

The mass and boundary stress-tensor as well as the thermodynamic quantities of a RNdS solutions are computed here by applying for the EM system the general formalism presented in refs. [19, 29] (see, also, ref. [18]).

We start by considering the path integral

$$\langle g_2, \Phi_2, S_2 | g_1, \Phi_1, S_1 \rangle = \int D\left[g, \Phi\right] \exp\left(iI\left[g, \Phi\right]\right), \tag{2.1}$$

which represents the amplitude to go from a state with metric and matter fields $[g_1, \Phi_1]$ on a surface S_1 to a state with metric and matter fields $[g_2, \Phi_2]$ on a surface S_2. The quantity $D[g, \Phi]$ is a measure on the space of all field configurations and $I[g, \Phi]$ is the action taken over all fields having the given values on the surfaces S_1 and S_2.

For asymptotically dS spacetimes we replace the surfaces S_1, S_2 with histories H_1, H_2 that have spacelike unit normals and are surfaces that form the timelike boundaries of a given spatial region. The amplitude (2.1) then is

$$\langle g_2, \Phi_2, H_2 | g_1, \Phi_1, H_1 \rangle = \int D\left[g, \Phi\right] \exp\left(iI\left[g, \Phi\right]\right) \tag{2.2}$$

and describes quantum correlations between differing histories $[g_1, \Phi_1]$ and $[g_2, \Phi_2]$ of metrics and matter fields, with the modulus squared of the amplitude yielding the correlation between two histories. Spacelike tubes at some initial and final times join the surfaces H_1, H_2, so that the boundary and interior region are compact. As these times approach past and future infinity one obtains the correlation between the complete histories, given by summing over all metric and matter field configurations that interpolate between them. The result does not depend on any special hypersurface between the hypersurfaces H_1 and H_2.

We decompose the action into the distinct parts

$$I = I_B + I_{\partial B} + I_{ct}, \tag{2.3}$$

where the bulk (I_B) and boundary ($I_{\partial B}$) terms are the usual ones, given by

$$I_B = \frac{1}{16\pi} \int_{\mathcal{M}} d^{n+2}x \, \sqrt{-g} \left(R - 2\Lambda + \mathcal{L}_M(\Phi)\right) \tag{2.4}$$

$$I_{\partial B} = -\frac{1}{8\pi} \int_{\partial\mathcal{M}^{\pm}} d^{n+1}x \, \sqrt{h^{\pm}} K^{\pm}, \tag{2.5}$$

where $\partial\mathcal{M}^\pm$ represents future/past infinity, and $\int_{\partial\mathcal{M}^\pm} = \int_{\partial\mathcal{M}^-}^{\partial\mathcal{M}^+}$ represents an integral over a future boundary minus a past boundary, with the respective metrics $h^\pm$ and extrinsic curvatures $K^\pm$. The quantity in (2.4) $\mathcal{L}_M(\Phi) = -F^2$ is the lagrangian for the Maxwell field. The bulk action is over the $(n+2)$-dimensional manifold $\mathcal{M}$, and the boundary action is the surface term necessary to ensure well-defined Euler-Lagrange equations.

For an asymptotically dS spacetime, the boundary $\partial\mathcal{M}$ will be a union of euclidean spatial boundaries at early and late times. The dS/CFT-inspired counterterm action I_{ct} is defined on these boundaries as suggested by analogy with the AdS/CFT correspondence, and its terms depend only on geometric invariants of these spacelike surfaces. It is universal in both the asymptotic AdS and dS cases [8, 19], and can be generated by an algorithmic procedure without reference to a background metric. In the dS case one obtains [19]

$$\begin{aligned} I_{ct} = -\frac{1}{8\pi}\int d^{n+1}x\sqrt{\gamma}\Big\{ &-\frac{n}{\ell} + \frac{\ell\Theta\,(n-2)}{2(n-1)}\mathsf{R} - \frac{\ell^3\Theta\,(n-4)}{2(n-1)^2(n-3)}\left(\mathsf{R}_{ij}\mathsf{R}^{ij} - \frac{n+1}{4n}\mathsf{R}^2\right) - \\ &- \frac{\ell^5\Theta\,(n-6)}{(n-1)^3(n-3)(n-5)}\times \\ &\times\left(\frac{3n+5}{4n}\mathsf{R}\mathsf{R}^{ij}\mathsf{R}_{ij} - \frac{(n+1)(n+3)}{16n^2}\mathsf{R}^3 - 2\mathsf{R}^{ij}\mathsf{R}^{kl}\mathsf{R}_{ijkl} - \right. \\ &\left. - \frac{n+1}{4n}\nabla_k\mathsf{R}\nabla^k\mathsf{R} + \nabla^k\mathsf{R}^{ij}\nabla_k\mathsf{R}_{ij}\right) + \cdots\Big\}, \end{aligned} \tag{2.6}$$

with R the curvature of the induced metric h_{ij} and, to assure consistency with previous works, we noted $H = 1/l$. The step-function $\Theta(x)$ is unity provided $x > 0$ and vanishes otherwise, thereby ensuring that the series only contains the terms necessary to cancel divergences and no more. In four ($n = 2$) dimensions, for example, only the first two terms appear, and only these are needed to cancel divergent behavior in $I_B + I_{\partial B}$ near past and future infinity.

The boundary metric can be written, at least locally, in ADM-like form

$$d\hat{s}^{\pm 2} = h^\pm_{ij}d\hat{x}^{\pm i}d\hat{x}^{\pm j} = N_\tau^{\pm 2}d\tau^2 + \sigma^\pm_{ab}\left(d\phi^{\pm a} + N^{\pm a}d\tau\right)\left(d\phi^{\pm b} + N^{\pm b}d\tau\right), \tag{2.7}$$

where N_τ and N^a are the lapse function and the shift vector respectively and the ϕ^a are the intrinsic coordinates on the closed surfaces Σ. It is clear that $\nabla_\mu\tau$ is a spacelike vector field that is the analytic continuation of a timelike vector field, the boundary metric(s) being euclidean (spacelike tube(s)) for asymptotically dS spacetimes.

Varying the action with respect to the boundary metric h_{ij}, we find the boundary stress-energy tensor for gravity

$$T^{\pm ij} = \frac{2}{\sqrt{h^\pm}}\frac{\delta I}{\delta h^\pm_{ij}} \tag{2.8}$$

(its explicit expression is given in ref. [19]). In this approach, the conserved quantities associated with a Killing vector $\xi^{\pm i}$ are given by

$$\mathfrak{Q}_\xi{}^\pm = \oint_{\Sigma^\pm} d^n\phi^\pm\sqrt{\sigma^\pm}n^{\pm i}T^\pm_{ij}\xi^{\pm j}\,, \tag{2.9}$$

JHEP01(2004)029

where $n^{\pm i}$ is an outward-pointing unit vector, normal to surfaces of constant τ. Here, ξ^i needs not be a bulk Killing vector; the quantity $\mathfrak{Q}_\xi$ will be conserved if ξ^i is a Killing vector only on the boundary. Physically, this means that a collection of observers, on the hypersurface with the induced metric h_{ij}, would all measure the same value of $\mathfrak{Q}_\xi$ provided this surface has an isometry generated by ξ^i. For any two histories the value of $\mathfrak{Q}$ is the same for each. This surface does not enclose anything [4]; rather it is the boundary of the class of histories that interpolate between H_1 and H_2. Consequently $\mathfrak{Q}$ is not associated with the class of histories that it bounds, but rather only with this boundary [29].

The conserved mass is defined to be

$$\mathfrak{M}^{\pm} = \oint_{\Sigma^{\pm}} d^n\phi^{\pm}\sqrt{\sigma^{\pm}}N_{\tau}^{\pm}n^{\pm i}n^{\pm j}T_{ij}^{\pm} , \tag{2.10}$$

provided $\partial/\partial\tau$ is a Killing vector on Σ; it is a function of the cosmological time $\mathcal{T}$. As $\mathcal{T}$ approaches positive or negative infinity, there is at least the notion of a conserved total mass $\mathfrak{M}^{\pm}$ for dS spacetime, since all asymptotically de Sitter spacetimes have an asymptotic isometry generated by $\partial/\partial\tau$.

An evaluation of the path integral may be carried out along the lines described in ref. [29]. Since the action is in general negative definite near past and future infinity (outside of a cosmological horizon), we analytically continue the coordinate orthogonal to the histories to complex values by an anticlockwise $\pi/2$-rotation of the axis normal to them. This generally imposes, from the regularity conditions, a periodicity β of this coordinate, which is the analogue of the Hawking temperature outside the cosmological horizon.

This renders the action pure imaginary, yielding a convergent path integral

$$Z' = \int e^{+\hat{I}} \tag{2.11}$$

since $\hat{I} < 0$. In the semi-classical approximation this will lead to $\ln Z' = +I_{cl}$. For a grand canonical ensemble with fixed temperature and fixed chemical potentials C_i and conserved charges μ_i (which are Φ and $\mathbf{Q}$ for a RNdS solution) we can write

$$W = \mathfrak{U} - TS - \mu_i C_i , \tag{2.12}$$

where W is the grand canonical (Gibbs) potential and $T = 1/\beta$. For a converging partition function, we have $W = I_{cl}/\beta$ and thus we find for the entropy of the system

$$S = \beta(\mathfrak{U} - \mu_i C_i) - I_{cl} . \tag{2.13}$$

2.1 Static coordinates

We now consider a charged black hole asymptotically dS spacetime in a static coordinate system. The corresponding line element reads

$$\begin{aligned} ds^2 &= \frac{dR^2}{F(R)} + R^2 d\Omega_n^2 - F(R)dT^2 , \\ F(R) &= 1 - \frac{2M}{R^{n-1}} + \frac{Q^2}{R^{2(n-1)}} - H^2R^2 , \end{aligned} \tag{2.14}$$

and gives a solution of the EM equations for a (pure electric) gauge potential

$$A = A_T dT = \left(\sqrt{\frac{n}{2(n-1)}} \frac{Q}{R^{n-1}} + \Phi \right) dT \,, \tag{2.15}$$

where Φ is a constant (to be fixed below). In the above expressions, M and Q are constants proportional to the gravitational mass $\mathfrak{M}$ and the total electric charge $\mathbf{Q}$, respectively.

A discussion of this solution appeared in refs. [27, 30]. Here we briefly review its basic properties. The metric has a curvature singularity at the origin $R = 0$. In general, there are three kinds of Killing horizon at the radii where $F(R)$ vanishes. Of interest are the outer black hole horizon at $R = R_+$ and the cosmological horizon $R = R_c \leq 1/H$ corresponding to the largest root of $F(R)$. The two horizons are not in thermal equilibrium because the time periods in the euclidean section required to avoid a conical singularity at both do not match in general. However, there are two families of RNdS solutions for which the black hole and cosmological horizon temperatures do match: the "lukewarm" black hole and charged Nariai black hole (see refs. [31, 32] for a detailed discussion).

The lukewarm solutions are characterized by the condition $Q^2 = M^2$, being stable endpoints of the evaporation process. In flat space, a black hole evaporates until it becomes extremal. An extremal black hole is in thermal equilibrium with radiation at any temperature. In dS spacetime the situation is different. That is, a black hole (with $Q^2 < M^2$) cannot become arbitrarily cold — the black hole will be stabilized by the Gibbons-Hawking radiation from the cosmological horizon. If $Q^2 > M^2$ the black holes are colder than the lukewarm solution and will absorb radiation from the cosmological horizon. As Q^2 increases relative to M^2 the inner and the outer black hole horizons eventually coincide, and the extremal (or "cold") RNdS black hole is obtained.

As the parameter M increases (relative to $|Q|$, with $M > 0$), the outer black hole and cosmological horizons move closer together. The charged Nariai solution is obtained when these horizons coincide at $R_h = R_c = R_+$; this is the largest charged asymptotically dS black hole. For a given Q, the charged Nariai black hole has the maximal mass M_N. Solving $F(R) = F'(R) = 0$ (so that R_h is a double root of $F(R)$) we obtain:

$$R_h^{n-1} = \frac{1}{2} M \, (n+1) \left(1 \pm \sqrt{1 - \frac{4nQ^2}{(n+1)^2 \, M^2}} \right)$$

for the event horizon. The charged Nariai solution and lukewarm solution meet for the critical value $Q^2 = M_c^2$, where

$$M_c^2 = \left(\frac{(n-1)^2}{H^2} \right)^{n-1} \frac{1}{n^{2n}} \,.$$

A black hole in dS spacetime is obtained for a range of mass parameter $M_{\min} \leq M \leq M_N$ (referred to as the "undermassive" case), where $M_{\min}$ is the mass of extremal RNdS. If these limits are exceeded, the metric (2.14) describes a naked singularity.

The computation of the mass, action and entropy of a RNdS black hole is a direct application of the method described in the previous section. We work outside of the cosmological horizon, where $F(R) < 0$. The topology of (2.14), for large constant R, is an euclidean cylinder $\mathbb{R} \times S^n$ and T is the coordinate along the cylinder. $\mathcal{I}^{\pm}$ are located outside the future/past cosmological horizons, where R is timelike and T is spacelike.

The gravitational mass/energy is the charge associated with the Killing vector $\partial/\partial T$ — now spacelike outside the cosmological horizon. The total energy found by using the counterterm prescription is

$$\mathfrak{U} = -\frac{\text{Area}(S^n)}{8\pi}\left(nM - \frac{n}{H^{n-1}}\frac{\Gamma(\frac{2p-1}{2})}{2\sqrt{\pi}\Gamma(p+1)}\delta_{2p,n+1}\right), \tag{2.16}$$

for p an integer, where $\text{Area}(S^n) = 2\pi^{\frac{n+1}{2}}/\Gamma((n+1)/2)$ is the area of a unit n-sphere. The second term is interpreted as the Casimir energy in the context of dS/CFT correspondence. We have explicitly checked this result up to $n = 7$. This reduces to the expression obtained in ref. [19] when $Q = 0$.

The total electric charge is computed by generalizing the methods of ref. [33] to the de Sitter case. Working again outside of the cosmological horizon, the electromagnetic field is

$$F_{RT} = -\sqrt{\frac{n(n-1)}{2}}\frac{Q}{R^n},$$

where R is timelike and T is spacelike. The induced metric $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects the electromagnetic field on a specific slice of the foliation. The electric field with respect to a slice $R = const.$ is $E_i = h_i^\mu F_{\mu\nu}u^\nu$ and the surface charge density at future/past infinity is

$$\rho_{\mathbf{Q}} = \sqrt{\sigma^{\pm}}n^{\pm i}E_i^{\pm},$$

where the vector $u^\nu = (\frac{1}{\sqrt{|F(R)|}}, 0, \vec{0})$ is normal to the induced metric $h_{\mu\nu}$ and $n^i = (0, \sqrt{|F(R)|}, \vec{0})$ is the (timelike) unit vector normal to hypersurface Σ.

Integrating the charge density over the hypersurface $\Sigma \equiv S^n$, we obtain the total electric charge at future/past infinity

$$\mathbf{Q} = -\frac{1}{4\pi}\oint_{\Sigma^{\pm}} d^n\phi^{\pm}\sqrt{\omega^{\pm}}n^{\pm i}h_i^\mu F_{\mu\nu}^{\pm}u^{\pm\nu} = 2Q\omega_n^{-1}\sqrt{\frac{2(n-1)}{n}}, \tag{2.17}$$

where $\omega_n = 16\pi/(n\text{Area}(S^n))$.

The interpretation of this charge is analogous to that for the other conserved quantities noted above. The charge $\mathbf{Q}$ is that measured by a collection of observers traversing a given history; conservation of charge implies that observers following a different history would measure the same value of the charge.

The computation of a finite action for RNdS black holes is only a slight generalization of the results presented in ref. [19]. Considering the EM equations of motion, we obtain the on shell bulk action

$$I_B = \frac{1}{16\pi}\int_{\mathcal{M}} d^{n+2}x\,\sqrt{-g}\left(\frac{4\Lambda}{n} - \frac{2}{n}F^2\right) \tag{2.18}$$

$$= \frac{(n+1)H^2}{8\pi}\int d^{n+1}x\int_{R_c}^{R_m} dR\frac{\sqrt{h}}{\sqrt{|F(R)|}} - \frac{1}{8\pi n}\int d^{n+1}x\int_{R_c}^{R_m} dR\frac{\sqrt{h}}{\sqrt{|F(R)|}}F^2\,. \tag{2.19}$$

In this expression the integration is from the cosmological horizon out to some R_m that will eventually be sent to infinity. We shall work here in the upper patch outside of the cosmological horizon in RNdS spacetime; results for the lower patch are obtained in a similar manner. The second term in the above relation is the usual electromagnetic field contribution. In computing it we keep A_T fixed at infinity, for a value of the constant appearing in eq. (2.15) corresponding to the electrostatic difference between the cosmological horizon and infinity

$$\Phi = -\frac{n\mathbf{Q}}{4(n-1)}\frac{\omega_n}{R_c^{n-1}},$$

which implies that the boundary term from the gauge field will vanish. A general expression for the rest of the action has been derived in ref. [19], for a generic $F(R)$.

In the large R limit we find the finite total action (after including the $I_{\partial B}$ and I_{ct} contributions)

$$I = \frac{\beta \mathrm{Area}(S^n)}{8\pi}\left[M + H^2 R_c^{n+1} - \frac{n}{H^{n-1}}\frac{\Gamma(\frac{2p-1}{2})}{2\sqrt{\pi}\Gamma(p+1)}\delta_{2p,n+1}\right] - \frac{1}{n}\beta\Phi\mathbf{Q},$$

where β is the periodicity of the coordinate T. This periodicity is usually fixed by Wick rotating and requesting the absence of conical singularities at $R = R_c$. The Wick rotation implies also a convergent path integral and allows us to compute (2.11) in the semi-classical approximation. Inserting these results in eqs. (2.12) and (2.13) we find a black hole entropy

$$S = \frac{\beta \mathrm{Area}(S^n)}{8\pi}\left(M(1-n) + H^2 R_c^{n+1}\right) - \frac{n-1}{n}\beta\Phi\mathbf{Q} = \frac{R_c^n \mathrm{Area}(S^n)}{4},$$

while the Hawking temperature of the cosmological horizon is

$$T_c = \frac{1}{\beta} = \frac{1}{2\pi R_c}\left((1-n)\frac{M}{R_c^{(n-1)}} + H^2 R_c^2 - (1-n)\frac{Q^2}{R_c^{2(n-1)}}\right).$$

The same expression is obtained by equating it to the surface gravity on the horizon. These quantities can be shown to be in accord with the first law of thermodynamics

$$dE = TdS + \Phi d\mathbf{Q}$$

for the cosmological horizon.

Subtracting off the vacuum energy contribution for the Casimir energy (which is nonzero only when n is odd) we find

$$\mathfrak{M} = -\frac{\mathrm{Area}(S^n)nM}{8\pi} = -\frac{R_c^{n-1}}{\omega_n}\left(1 - H^2 R_c^2 + \frac{Q^2}{R_c^{2(n-1)}}\right), \tag{2.20}$$

for the gravitational mass, measured at the far past or far future boundary of dS space. Note that it is negative provided $M > 0$, consistent with the expectation [18] that pure dS spacetime has the largest mass for a singularity-free spacetimes of this topology. If there is a CFT dual to RNdS, this mass translates into the energy of the dual CFT living on a euclidean cylinder $\mathbb{R}\times S^n$. The existence of a macroscopic scale in the system explains the appearance of the Casimir term (proportional with the central charge in two dimensions) in the expression of the gravitational energy (2.16).

As in the AdS/CFT correspondence, the metric of the manifold on which the *putative* dual CFT resides is defined by

$$\gamma_{ij} = \lim_{R\to\infty} \frac{1}{H^2R^2} h_{ij} \, .$$

The dual field theory's stress tensor, τ^i_k, is related to the boundary stress tensor by the rescaling [34]

$$\sqrt{\gamma}\gamma^{ij}\tau_{jk} = \lim_{R\to\infty} \sqrt{h}h^{ik}T_{jk} \, .$$

The geometry of the manifold on which the dual CFT resides is given by the cylinder metric

$$ds^2 = \gamma_{ij}dx^i dx^j = dT^2 + \frac{1}{H^2}d\Omega^2_n \, . \tag{2.21}$$

The CFT's stress tensor is

$$\tau^i_j = -\frac{1}{8\pi}(M - \frac{1}{H^{n-1}}\,\frac{\Gamma(\frac{2p-1}{2})}{2\sqrt{\pi}\Gamma(p+1)}\delta_{2p,n+1})(-(n+1)u^iu_j + \delta^i_j)\, , \tag{2.22}$$

where $u^i = \delta^i_T$. This tensor is finite, covariantly conserved and manifestly traceless. The trace anomaly indicates the quantum breaking of conformal invariance in curved spaces and is proportional to both the curvature and central charge in two-dimensions. The sphere S^n has a scale, $1/H^2$, but the conformal invariance of the theory is preserved since this scale enters in a conformally invariant way in the metric itself [35]. Consequently, we obtain a traceless stress tensor (2.22).

Working outside the cosmological horizon, there is no manifestation of the black hole horizon in the expression of the CFT's stress tensor. We will see in section 4 that, using planar coordinates which cover more of the bulk manifold, the black holes will appear like punctures in the dual theory corresponding to divergences in the dual stress tensor.

Verlinde [36] generalized the Cardy formula, that counts the quantum states of a two-dimensional CFT, to any dimension. This result, now commonly referred to as the Cardy-Verlinde formula, is supposed to be a formula yielding the entropy for a CFT in any dimension. As discussed in ref. [37] (see also refs. [38, 39]), the entropy of the cosmological horizon of a RNdS solution can be rewritten in the form of the Cardy-Verlinde formula:

$$S = \frac{2\pi}{nH}\sqrt{|E_c|(2(\mathfrak{M} - E_q) - E_c)}\, , \tag{2.23}$$

where E_q is the energy of the U(1) field

$$E_q = \frac{1}{2}\Phi\mathbf{Q} = -\frac{Q^2}{\omega_n R_c^{n-1}}$$

and the Casimir energy E_c, defined in this context as the violation of the Euler identity $E_c = (n+1)\mathfrak{M} - nTS - n\Phi\mathbf{Q}$, is

$$E_c = -\frac{2nR_c^{n-1}\mathrm{Area}(S^n)}{16\pi}\, . \tag{2.24}$$

These results are usually interpreted as providing support for the dS/CFT correspondence. Note that, unlike the AdS/CFT case, the Casimir energy is negative, indicating the non-unitarity of the dual CFT [15].

Similar thermodynamic quantities can be defined also for the black hole horizon, while the total entropy of the RNdS spacetime is believed to be the sum of entropies of the black hole and cosmological horizons. Since the total entropy of the RNdS spacetime is smaller than that of empty dS spacetime, the black hole is made by borrowing degrees of freedom from the horizon of empty dS spacetime and freezing them into special configurations.

As observed in ref. [37], if one uses the BBM definition of the mass, the black hole entropy cannot be rewritten in a Cardy-Verlinde form. However, when the Abbot-Deser prescription for the total mass is used (which gives a positive gravitational mass), the black hole entropy can also be written in a form similar to eq. (2.23).

We conclude that the CFT on $\mathcal{I}^+$ accounts for the entropy of the cosmological horizon only, in accord with the fact that the thermodynamic quantities are measured by observers outside the cosmological horizon.

2.2 Planar coordinates

Although the line element (2.14) takes a simple form in a static coordinate system, the expansion of the universe is not manifest. Furthermore, the static coordinates (T, R) break down at the Killing horizons. However RNdS black holes admit a simple expression in planar coordinates

$$\begin{aligned} ds^2 &= a^2 U^{\frac{2}{n-1}}(dr^2 + r^2 d\Omega_n^2) - \frac{V^2}{U^2}dt^2\,, \\ a &= e^{Ht}\,, \qquad V = 1 - \frac{\alpha}{\rho^2}\,, \qquad U = 1 + \frac{M}{\rho} + \frac{\alpha}{\rho^2}\,, \end{aligned} \tag{2.25}$$

where $\rho = (ar)^{n-1}$ and $\alpha = (M^2 - Q^2)/4$.

For $n > 2$, the metric (2.25) generalizes the four dimensional solutions discussed in ref. [27]. For electrically charged black holes, the only nonvanishing component of the U(1) potential is:

$$A_t = Q\sqrt{\frac{n(n-1)}{2}}\int a^{1-n} V (U^2 r^n)^{-1} dr\,. \tag{2.26}$$

Using planar coordinates has the disadvantage that the manifest time-translation symmetry is broken and the charts that cover the horizons are highly distorted. The coordinate transformation relating the static and planar metrics is

$$R = arU^{\frac{1}{n-1}}\,, \qquad t = T + h(R)\,, \qquad \text{with} \qquad h'(R) = -\frac{HR}{F(R)\sqrt{F(R) + H^2R^2}}\,. \tag{2.27}$$

The time-dependence enters in a natural manner in this solution and for large r, the dS metric in planar coordinates is approached. We note that the case $Q = 0$ is just the McVittie solution describing a Schwarzschild black hole embedded in $(n+2)$-dimensional de Sitter spacetime [25]. A discussion of the global structure of the line element (2.25), as well as the the coordinate transformation (2.27) appears in ref. [27] for $n = 2$. It is

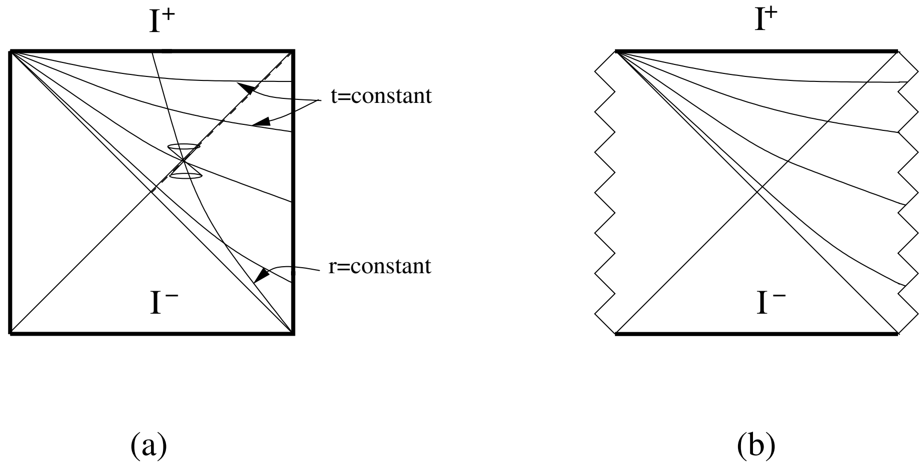


Figure 1: Carter-Penrose diagram for pure dS spacetime (a) and for the "overmassive" case (b). Singularities are represented by wavy lines.

straightforward to show that these general features remain valid for $n > 2$, where the analysis depends crucially on the sign of H. Analogues with pure dS spacetime (see figure 1a), cosmologically expanding ($H > 0$) and contracting ($H < 0$) coordinate systems also exist in RNdS spacetime, being centered only about the black hole. Finally we note that when $M > M_N$ (the "overmassive" case) the global structure is strongly modified as in figure 1b, and a timelike naked singularity appears.

Although the singularity is naked, this is not a violation of cosmic censorship. The reason is that we start with bad singular initial conditions that imply the naked singularity exists at any time. The outer black hole and the cosmological horizons have disappeared. However the inner black hole horizon can be interpreted as a cosmological one, separating the two naked singularities at antipodal regions of a background dS spacetime.

The geometry (2.25) is preserved by the transformation

$$t \to t + \lambda\,, \qquad r \to e^{-\lambda H} r\,. \tag{2.28}$$

In the above relation, the first term generates time evolution in the bulk gravity theory, while the second term generates scale transformations in the boundary theory. The Killing vector associated with this symmetry reads

$$\xi = -Hr\frac{\partial}{\partial r} + \frac{\partial}{\partial t} \tag{2.29}$$

and is not globally timelike. $\mathcal{I}^{\pm}$ are now approached for large $|t|$, while the boundary topology is R^{n+1}. For $H > 0$, we take the euclidean CFT to live at future infinity $\mathcal{I}^+$ on the space parametrized by x^i. From the UV/IR correspondence, an object at time t in dS space corresponds to an excitation of scale size $\delta x = e^{Ht}$ in the CFT. The CFT resides on the euclidean line element

$$ds^2 = \gamma_{ij} dx^i dx^j = dr^2 + r^2 d\Omega_n^2\,. \tag{2.30}$$

A straightforward calculation following the BBM prescription gives the boundary stress-tensor associated with the line-element (2.25):

$$T_{rr} = -\frac{1}{8\pi H}\frac{n}{n-1}\frac{U_{,r}}{rU}\left(1+\frac{r}{2(n-1)}\frac{U_{,r}}{U}\right) = \frac{M}{8\pi H r^2}\frac{n}{(ar)^{n-1}} + O\left(\frac{1}{a^n}\right), \tag{2.31}$$
$$T_{ab} = -\frac{r^2\omega_{ab}}{8\pi H}\frac{1}{n-1}\left(-\frac{n}{n-1}\left(\frac{U_{,r}}{U}\right)^2 + (n-1)\frac{U_{,r}}{rU}+\frac{U_{,rr}}{U}\right) = -\frac{M}{8\pi H}\frac{\omega_{ab}}{(ar)^{n-1}}+O\left(\frac{1}{a^n}\right).$$

We arrived at the above result using the fact that for the planar slicing the extrinsic curvature is proportional with the intrinsic metric for each slice, $K_{ij} = -Hh_{ij}$, and the first three terms in the boundary stress tensor exactly cancel.

The stress tensor of the dual field theory has the simple form

$$\tau_r^r = \frac{M}{8\pi H}\frac{n}{r^{n+1}}, \qquad \tau_b^a = -\delta_a^b\frac{M}{8\pi H}\frac{1}{r^{n+1}}. \tag{2.32}$$

This tensor is covariantly conserved and manifestly traceless, as expected for a conformal field theory on a manifold with zero curvature. The presence of the divergent factor $1/r$ in the above expression reflects the inhomogeneity of the bulk geometry.

The quantity we need is the mass of the black hole as an excitation in dS_{n+2}. The mass is the charge associated with the Killing vector (2.29), measured at the far past or far future (depending on the sign of H):

$$\mathfrak{U} = \mathfrak{M} = -\frac{\mathrm{Area}(S^n)nM}{8\pi},$$

yielding a value for the energy of the dual CFT which agrees with (2.20). The negative sign implies that the black hole lowers the total bulk energy with respect to the total energy of the pure dS space, which is zero in planar coordinates. The absence of a macroscopic scale for a boundary with topology R^{n+1}, unlike the static-coordinate case when the boundary topology is $\mathbb{R}\times S^n$, leads to the absence of a Casimir contribution [21].

The norm of the Killing vector vanishes at the horizons, which are again the dS horizon r_c and the outer black hole horizon r_+, being located at the solutions of the equation $\xi_k\xi^k = 0$. However, by using the coordinate transformation (2.27) we find that $\xi_k\xi^k = 0$ corresponds to $F(R) = 0$, which is the equation for the position of a horizon in static coordinates. Thus, although the location of a horizon is time-dependent in planar coordinates, the product $r_h a$ is constant. The event horizon radius in static coordinates R_h is related to $\rho_h = (ar_h)^{n-1}$ through the relation $M + \rho_h + \alpha/\rho_h = R_h^{n-1}$.

The thermodynamic properties of the RNdS black hole in planar coordinates can also be discussed, at least formally. The concept of assigning a temperature to dS space is well-defined only in the static patches. However, we can assign a Hawking temperature to these event horizons by using the standard relation $T = |\kappa|/2\pi$, where κ is the surface gravity of the horizons, computed from

$$\frac{1}{2}\nabla_k(\xi_i\xi^i) = -\kappa\xi_k\,.$$

The values of the Hawking temperature for both the black hole and cosmological horizons are the same in both coordinate systems. We can prove that by using the properties of the coordinate transformation (2.27)

$$\left.\frac{dF(r(R,T))}{dr}\right|_{r=r_c} = \left.\frac{dF(R)}{dR}\frac{dR}{dr}\right|_{r=r_c}, \qquad \frac{dR}{dr} = \frac{VR}{Ur}\,.$$

Furthermore, the event horizon area is the same in both coordinate systems since $A = (r_h a U^{\frac{1}{n-1}})^n \mathrm{Area}(S^n)$, and from eq. (2.27), $A = R_h^n \mathrm{Area}(S^n)$.

Thus, we are motivated to associate an entropy $S = A/4$ with the horizons of the cosmological RNdS solution (2.25). Following Bousso's formulation of the holographic principle [40], an entropy bound for empty dS space in planar coordinates has been discussed in ref. [41]. However, similar arguments can be used here also. The entropy contained at time t in a spatial ball of arbitrary radius $x^i x^i \le \tilde{r}^2$ is bounded by [40],

$$S = \frac{\text{surface area of ball}}{4} = \frac{\mathrm{Area}(S^n)}{4}(a\tilde{r}U^{\frac{1}{n-1}})^n\,.$$

For $\tilde{r} = e^{-Ht}U^{\frac{1}{1-n}}R_c$, i.e. when the surface ball coincides with the cosmological horizon, this is exactly the entropy of the RNdS space in static coordinates.

Thus, although the horizon and entropy in dS space have an obvious observer dependence, thermodynamic quantities defined in this way are the same as those in the case of static coordinates.

We can further demonstrate that the dS horizon entropy adopts the Cardy-Verlinde-like form (2.23). In deriving the expression of the chemical potential Φ we make use of the gauge properties of the U(1) field, and similar to the static case we express all the thermodynamic quantities in terms of $(Q,\ R_c,\ H)$. Therefore we find the same general picture for two different patches, with different classes of observers. We can interpret this result as providing support for the dS/CFT correspondence, since the general features of the CFT dual to a RNdS black hole should not depend on the dS slicing choice.

3. Multi-black hole solutions

A case of particular interest is $Q^2 = M^2$. As originally found by Kastor and Traschen in four dimensions [28], this extremal RNdS metric admits a generalization to N black holes, with the line-element [42]

$$\begin{aligned} ds^2 &= a^2\Omega^{\frac{2}{n-1}}((dx^1)^2 + \cdots + (dx^{n+1})^2) - \Omega^{-2}dt^2\,, \\ \Omega &= 1 + \frac{1}{a^{n-1}}\sum_{A=1}^{N}\frac{M_A}{|r-r_A|^{n-1}}\,, \qquad a(t) = e^{Ht}\,. \end{aligned} \tag{3.1}$$

Here $|r - r_A|$ denotes the euclidean distance between the field point r and the fixed location r_A in a euclidean space of cosmological coordinates: $|r - r_A| = (\sum_{k=1}^{n+1}(x^k - x_A^k)^2)^{1/2}$. The black hole locations, r_A, are arbitrary, while the Maxwell one-form is given by $A = A_t dt = \Omega\sqrt{n/(2(n-1))}dt$.

The KT solution is time dependent and has no symmetries in general. A generalization of this solution to include a dilaton was given in ref. [43].

In the limit of vanishing cosmological constant, the KT solution reduces to the well-known static Majumdar-Papapetrou solution [44]. In contrast, the black holes with $\Lambda > 0$ are highly dynamical: they ignore one another and follow natural trajectories in the background of dS space. For $H < 0$, the KT solution describes a system of "incoming" charged black holes (all with the same sign of charge), which collide and coalesce for a certain range of parameters, providing a new arena in which to test cosmic censorship. The solutions describing the time-reversed situation (a system of "outgoing" white holes, oppositely charged) is obtained by changing the sign of H [28, 45]. The "incoming" set is described by cosmological coordinates that include $\mathcal{I}^-$, whereas for the "outgoing" set, $\mathcal{I}^+$ is included.

The surfaces of constant t are spacelike everywhere and the boundary metric, approached for large t, is $ds^2 = a^2\Omega^{\frac{2}{n-1}}((dx^1)^2 + \cdots + (dx^{n+1})^2)$. However, a multi-black hole solution has no isometry of the form (2.28) and there is no quasilocal conserved charge associated with M, nor any well-defined quasilocal thermodynamic quantities.

A computation of the boundary stress tensor is still possible, with the result

$$T_{ij} = -\frac{1}{8\pi H}\frac{1}{n-1}\left(\frac{n}{n-1}\frac{\Omega_{,i}\Omega_{,j}}{\Omega^2} - \frac{\Omega_{,ij}}{\Omega} + \delta_{ij}\left(\frac{n-4}{2}\frac{\Omega_{,k}\Omega_{,k}}{\Omega^2} + \frac{\Omega_{,kk}}{\Omega}\right)\right).$$

The corresponding stress tensor of the *presumed* dual field theory is covariantly conserved, traceless and has the form

$$\tau^{ij} = -\frac{1}{8\pi H}\sum_{A=1}^{N}\frac{M_A}{|r-r_A|^{n+1}}\left(\delta^{ij} - (n+1)\frac{(x^i-x^i_A)(x^j-x^j_A)}{|r-r_A|^2}\right), \qquad (3.2)$$

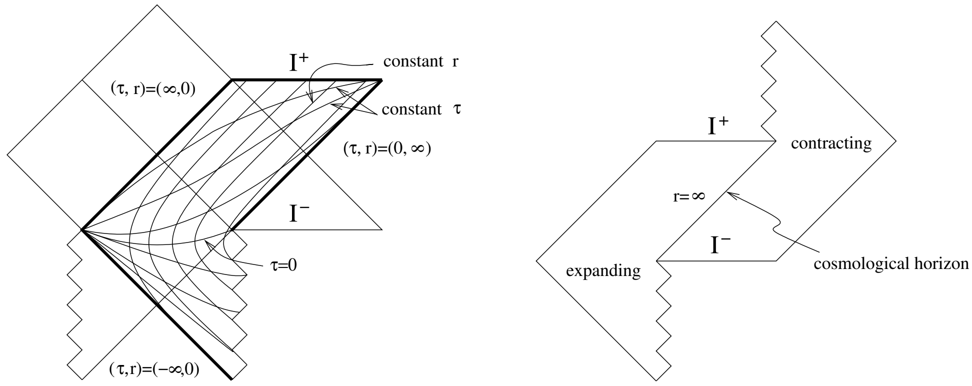
diverging again at the black hole locations. Since the isometry (2.28) is recovered for $r \gg r_A$, we can define a mass only asymptotically. Thus, in the limit of large r, and for r_A in a compact region of euclidean coordinate space, the $N-$black hole solution approaches the solution (2.25) with the mass parameter $M = \sum M_A$ and the total gravitational mass is the sum of the individual masses

$$\mathfrak{M} = -\frac{\mathrm{Area}(S^n)n}{8\pi}\sum M_A = \sum \mathfrak{M}_A\,.$$

Another case of interest is when $|r - r_A|$ is small, for fixed A. Let us choose the origin so that $r_A = 0$. The conformal factor of the boundary metric

$$a(t)^2\Omega^{\frac{2}{n-1}} \approx \left(a(t)^{n-1} + \frac{M_A}{|r|^{n-1}} + \sum_{B\neq A}\frac{M_B}{|r_B|^{n-1}}\right)^{\frac{2}{n-1}} = \left(a(t)^{n-1} + \frac{M_A}{|r|^{n-1}} + \mathrm{const.}\right)^{\frac{2}{n-1}}$$

differs by a constant from the conformal factor of a metric that describes just a black hole with mass parameter M_A. It is clear that an observer on a boundary near to the A^{th} black hole, will measure a gravitational mass $\mathfrak{M} = \mathfrak{M}_A$ if the other black holes are far enough away (i.e. if r_B is sufficiently large).



(a) (b)

Figure 2: Carter-Penrose diagram for an undermassive RNdS black hole in the case Q=M at the location $r_A = 0$. The region covered by the expanding chart is shown in a, the thick lines representing the boundaries of the chart and the wavy lines singularities. An expanding and a contracting patch are shown in b, but the figure can be repetead indefinitely in the horizontal and vertical direction, yielding a spacetime that is spacelike and timelike periodic.

Unlike (2.22), the CFT's stress tensor in planar coordinates — see eqs. (2.32) and (3.2) — has divergences. Consider the meaning[1] of these divergences in the $Q = M$ case for $n = 2$; these interpretations are also valid for general n. An analysis of these configurations similar to the single-black hole case appears difficult — their properties are better understood by using a coordinate system with $\tau = e^{Ht}/H$ and continuing τ to negative values (leaving unchanged the form of the CFT stress tensor (3.2)). The metric becomes [27, 45]:

$$ds^2 = -\frac{d\tau^2}{U(\tau,r)^2} + U(\tau,r)^2 dr \cdot dr\,, \qquad U(\tau,r) = H\tau + \sum_A \frac{M_A}{|r - r_A|}\,.$$

Let us work in expanding patch ($H > 0$), that covers the expanding cosmological region and part of the white hole. This is shown explicitly in figure 2a. The black hole is "undermassive" ($M < M_N$), so all three horizons (inner and outer black hole horizons and cosmological horizon) are present. The boundaries of the chart are given by: the black hole event horizon at ($\tau = +\infty, r = r_A$), the inner horizon at ($\tau = -\infty, r = r_A$), the singularity for $U = 0$ and the $\mathcal{I}^+$ at ($\tau = +\infty, r =$ finite). The maximal analytic extension [27], for one black hole, can be obtained by gluing an expanding and a contracting cosmological patch at the cosmological horizon ($\tau = 0, r = +\infty$), as illustrated in figure 2b.

Near r_A the metric describes a cylindrical geometry resembling the infinite throat of the asymptotically flat RN. The slice $\tau = 0$ is regular and has a cylindrical form everywhere. In fact, we see that a singularity appears at $r = +\infty$ (corresponding to $\mathrm{U}(0, r = +\infty) = 0$). The singularity cuts off more and more of the cylinder, as τ is decreasing to negative values, and at $\tau = -\infty$ the singularity surrounds the throat at $r = r_A$. The slices given by $\tau > 0$

[1] Here we recapitulate the discussion from ref. [45] (see, also, ref. [46]), in the light of the dS/CFT correspondence.

are non-singular since $\mathrm{U}(\tau, r) > 0$ in this case. It is clear now that $\mathcal{I}^+$ is an asymptotically flat euclidean surface, with the cylindrical form of an infinite throat near $r = r_A$. The dual CFT is defined on $\mathcal{I}^+$, that is a plane with the "point" $r = r_A$ removed, and the black hole appears like a puncture in the dual theory.

We can extend this discussion to the KT multi black-hole solution case. We showed that in the limit of large r the multi-black hole solution resembles the RNdS solution with the gravitational mass given by the sum of individual masses. We can still glue expanding and contracting patches at $r = +\infty$, but this can be done with varying degrees of smoothness. This means that there is not a unique analytic extension.[2] $\mathcal{I}^+$ is again a non-singular and asymptotically flat at $r = +\infty$ with the form of an infinite throat around each "point" r_A, as in figure 3.

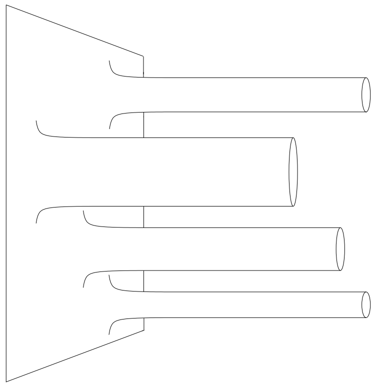


Figure 3: The surface $\mathcal{I}^+$ of an expanding KT spacetime. The missing "points" represent disjoint boundary components. In the expanding patch the black holes remain separately for all times.

The interpretation of the divergences in the stress tensor of the CFT is that the black holes appear as punctures in the dual theory.

4. RG flow and c-function

A remarkable property of the AdS/CFT correspondence is that it works even far from the conformal regime: supergravity describes an $N = 1$ supersymmetric renormalization group (RG) flow and there is a c-function that is monotonic decreasing from the ultraviolet (UV) regime (large radii in AdS space) to the infrared (IR) regime (small radii in the AdS space) of the dual CFT (see, e.g., refs. [47]–[49]). This result is consistent with the interpretation of the radial coordinate of AdS space as a energy scale: the CFT on the boundary is scale invariant and the QFT has a scale (energy) μ. The RG "trajectory" allows us to define the UV limit ($\mu \to \infty$) and the IR limit ($\mu \to 0$) of a given QFT. In the AdS/CFT context, the CFT on the boundary is the UV fixed point of a QFT in the bulk. The central charge counts the number of massless degrees of freedom in the CFT (it counts the ways in which energy can be transmitted). The coarse graining of a quantum field theory removes the information about small scales, in other words there is a gradual loss of non-scale invariant degrees of freedom.

In ref. [47], the interior holographic duals for AdS were investigated by adopting a wilsonian RG perspective. That is, to foliate AdS space with surfaces of constant radial coordinate, choose a slice that enclosed a specific volume and perform the bulk path integral over the excluded volume. In this way, a subset of the system is described by "integrating out" the excluded degrees of freedom. Different foliations of the spacetime lead to different RG flows and in general there is no requirement that a field should be coarsened uniformly.

[2] In the final section we comment on the physical interpretation of this lack of smoothness and on cosmic censorship.

In what follows,[3] we use these ideas to understand how the RG flow is modified when a charged black hole exists in dS space and also how it is changed by different foliations.[4] In analogy with the AdS/CFT case, we can interpret the time evolution in asymptotically dS spaces as a RG flow [18, 50]. A step further was made in ref. [51], where a c-function, which can be evaluated on each slice of some foliation, was defined. The idea is to associate an effective cosmological constant

$$\Lambda_{\text{eff}} = G_{\mu\nu} n^{\mu} n^{\nu} = \rho$$

to any slice that can be embedded in inflationary dS space; here n^{μ} is the unit normal vector to a constant time slice and ρ is the energy density on a constant time hypersurface. By dimensional analysis and imposing the c-function to be a function of this effective cosmological constant, one finds:

$$c \sim \frac{1}{\Lambda_{\text{eff}}^{n/2}} = \rho^{-\frac{n}{2}} . \tag{4.1}$$

In ref. [52], the c-function is geometrically interpreted as the area of the apparent cosmological horizon in Planck units. The generalized dS c-theorem states that in a contracting patch of dS spacetime, the RG flows toward the infrared and in an expanding spacetime, it flows toward the ultraviolet.

To compute Λ_{eff} we make use of the Einstein equations

$$G_{\mu}^{\nu} + \Lambda \delta_{\mu}^{\nu} = 2T_{\mu}^{\nu} ,$$

where $T_{\mu}^{\nu} = F_{\mu\alpha}F^{\alpha\nu} - \frac{1}{4}F^2\delta_{\mu}^{\nu}$ is the energy-momentum tensor of the gauge field. In static coordinates, the only nonzero component of the gauge field is A_T, and so

$$F^2 = 2F_{RT}F^{RT} = -\frac{n^2\omega_n}{8R^{2n}}\mathbf{Q}^2 .$$

However, this is an invariant, valid also in planar coordinates and we obtain $T_t^t = T_T^T = \frac{1}{4}F^2$. Thus, the effective cosmological constant in static coordinates is given by

$$\Lambda_{\text{eff}} = -G_T^T = \Lambda - 2T_T^T = \Lambda - \frac{1}{2}F^2 = \Lambda + \frac{n^2\omega_n}{16R^{2n}}\mathbf{Q}^2 \tag{4.2}$$

and is larger in the bulk than on the boundary ($R \to +\infty$). The corresponding c-function is decreasing from the (conformal) boundary to the bulk and has finite values at $\mathcal{I}^{\pm}$ and at the cosmological horizon [51]. For $Q = 0$, the Schwarzschild-dS vacuum solution is obtained and the RG flow is trivial corresponding to a constant $\Lambda_{\text{eff}} = \Lambda$.

In planar coordinates (r, t), the effective cosmological constant is given by

$$\Lambda_{\text{eff}} = -G_t^t = \Lambda - 2T_t^t = \Lambda - \frac{1}{2}F^2 . \tag{4.3}$$

[3] We thank Rob Myers for suggestions and discussions on this section.

[4] We note that holographic RG flows have been used in the context of the dS/CFT correspondence to calculate the holographic conformal anomaly [53].

It is easy to demonstrate that

$$\frac{\partial T_t^t}{\partial t} = 4n(n-1)^2\mathbf{Q}^2\frac{1}{R^{2n+1}}\frac{\partial R}{\partial t}\,, \qquad \frac{\partial R}{\partial t} = \frac{HVR}{U}\,. \tag{4.4}$$

In the case $Q = M$, the sign of $\frac{\partial T_t^t}{\partial t}$ is given by the sign of H. Therefore, in a contracting (expanding) patch $H < 0$ ($H > 0$) the effective cosmological constant is a monotonically increasing (decreasing) function of t and the corresponding c-function is a monotonically decreasing (increasing) function of t. As we expected, the RG flow in planar coordinates is not uniform because the induced metric on a slice $t = const.$ has also an r dependence. This resembles the lattice field theory case where meshes of different sizes are used in different regions [47].

5. From dS to AdS space

In static coordinates, we can formally "Wick rotate" from a positive to a negative cosmological constant by using the analytic continuation $H \to -iH$. Together with $T \to iT$ this generates a solution in the euclidean AdS space (note also that, for a real EM solution we must analytically continue the electric charge $Q \to iq$). However there are different analytic continuations that generate solutions with lorentzian signatures. As discussed in ref. [54] in the AdS/CFT context, the continuation $\theta \to \pi/2 + i\tau$ gives a bubble spacetime with a line element

$$\begin{aligned} ds^2 &= \frac{dR^2}{F(R)} + R^2\left(-d\tau^2 + \cosh^2\tau d\Omega_{n-1}^2\right) + F(R)dT^2\,, \\ F(R) &= 1 - \frac{2M}{R^{n-1}} - \frac{q^2}{R^{2(n-1)}} + H^2R^2\,. \end{aligned} \tag{5.1}$$

Bubble spacetimes have previously been considered by various authors as examples of time-dependent backgrounds with negative cosmological constant [54]-[59]. Typically, these spacetimes are obtained by double analytic continuations of asymptotically AdS (or flat) black holes.

However, in special cases, similar constructions can be obtained starting with dS solutions [58]. For the RNdS solution in cosmological coordinates (2.25), the transformation $t \to i\chi$, $H \to -iH$, $Q \to iq$ generates a euclidean solution of the EM equations with negative cosmological constant. For $M = Q = 0$ this gives the Poincare patch of AdS_{n+2}.

A simple analytic continuation yielding a lorentzian line-element is again $\theta \to \pi/2 + i\tau$. The resulting spacetime is

$$\begin{aligned} ds^2 &= \frac{V^2}{U^2}d\chi^2 + a^2U^{\frac{2}{n-1}}\left(dr^2 + r^2\left(-d\tau^2 + \cosh^2\tau d\Omega_{n-1}^2\right)\right) \\ V &= 1 - \frac{M^2+q^2}{4(ar)^{2(n-1)}}\,, \qquad U = 1 + \frac{M}{(ar)^{n-1}} + \frac{M^2+q^2}{4(ar)^{2(n-1)}}\,, \qquad a = e^{H\chi}\,, \end{aligned} \tag{5.2}$$

which is a solution of the EM equations with negative cosmological constant. The coordinate transformation between the known RN-AdS bubble solution (5.1) and the line element (5.2) reads

$$R = arU^{\frac{1}{n-1}}\,, \qquad t = T + h(R)\,, \qquad \text{with} \qquad h'(R) = -\frac{HR}{F(R)\sqrt{F(R) - H^2R^2}}\,.$$

The spacetime (5.2) describes a "bubble" in an asymptotically anti-de Sitter spacetime, which contracts from infinite size at $\tau \to -\infty$ to a minumum size as $\tau = 0$ and then expands back out to infinite size as $\tau \to \infty$. One can prove that the metric (5.2) is smooth as $(ar)^{2(n-1)} \to \alpha$ (or $V \to 0$). Consequently there is no intrinsic periodicity of the coordinate χ; indeed there cannot be or else the metric will not be single-valued. We can understand this by observing that the region around R_h (with $F(R_h) = 0$) in eq. (5.1) is not covered by the coordinate system (r, χ) (since $V^2/U^2 = F(R) - H^2R^2$). Since χ is not periodic, the bubble does not close back on itself — in this sense it is not really a bubble. However the metric (5.2) should be a valid background for string theory.

The boundary CFT metric obtained as χ becomes positively infinite is the $(n+1)$ dimensional flat spacetime metric written in unusual coordinates

$$ds^2 = dr^2 + r^2\left(-d\tau^2 + \cosh^2\tau d\Omega^2_{n-1}\right)\,, \tag{5.3}$$

and is similar to the metric obtained as $\chi \to -\infty$ (the conformal factor $1/r^4$ obtained in this case can be thrown away by a redefinition of the "radial" coordinate). Calculation of the boundary stress tensor for the spacetime (5.2) is straightforward. If we assume that the AdS/CFT correspondence can be extended to such asymptotically locally AdS spaces, the dual description of this spacetime will be given by some sort of the SYM theory on R^{n+1}.

Another interesting case is again the limit $Q^2 = M^2$. For extremal RNdS black holes in planar coordinates, a general analytic continuation yielding lorentzian AdS solutions with the right signature seems impossible. However $Q^2 = M^2$ does not correspond to an extremal black hole when a cosmological constant is present; hence, at least in four dimensions, we can generate new solutions of the EM equations on the euclidean section, with $\Lambda < 0$. An AdS counterpart for the KT solution (3.1) is obtained by using the Wick rotations $t \to i\chi$, $H \to -iH$. Here we should consider magnetically-charged black holes in order to find real solutions. This corresponds to multi-center Einstein-Maxwell instantons in AdS_4.

The two-center solution takes a particularly simple form. We start by writing the $N = 2$ KT solution in a prolate spheroidal coordinate system [60]. Without loss of generality, the masses are placed at $(0,0,1)$ and $(0,0,-1)$ respectively. Thus, by defining

$$x^1 = \sinh\psi\sin\theta\cos\varphi\,, \qquad x^2 = \sinh\psi\sin\theta\sin\varphi\,, \qquad x^1 = \cosh\psi\cos\theta\,, \tag{5.4}$$

the two-center KT metric becomes

$$ds^2 = -U^{-2}dt^2 + e^{2Ht}U^2\left(P(d\psi^2 + d\theta^2) + \sinh^2\psi\sin^2\theta d\varphi^2\right)\,, \tag{5.5}$$

where

$$U = 1 + \frac{We^{-Ht}}{P}\,, \qquad P = (\sinh^2\psi + \sin^2\theta)\,, \qquad W = (M_1 + M_2)\cosh\psi + (M_1 - M_2)\cos\theta\,.$$

We consider here a purely magnetic potential $A_\varphi = 4\cos\theta\sinh^2\psi/(\cos 2\theta - \cosh 2\psi)$. The corresponding two-center euclidean AdS_4 solution of EM equations is found by analytically continuing $t \to i\chi,\quad H \to -iH$ and reads

$$ds^2 = \left(1 + \frac{e^{-H\chi}W}{P}\right)^{-2} d\chi^2 + e^{2H\chi}\left(1 + \frac{e^{-H\chi}W}{P}\right)^2 \left(P(d\psi^2 + d\theta^2) + \sinh^2\psi\sin^2\theta d\varphi^2\right), \tag{5.6}$$

with P, W, A_φ still given by the above relations. The boundary CFT metric obtained in this case for large positive χ corresponds again to an unusual parametrization of the flat space

$$ds^2 = P(d\psi^2 + d\theta^2) + \sinh^2\psi\sin^2\theta d\varphi^2\,, \tag{5.7}$$

while in the limit $\chi \to -\infty$ the above metric gains a conformal factor $(W/P)^2$.

A more detailed discussion of these solutions will be presented elsewhere.

6. Discussion

Unlike the AdS/CFT correspondence, the conjectured dS/CFT correspondence is far from being understood. A first step toward this understanding is to study various configurations in asymptotically dS space. To this end we have studied some properties of the RNdS black hole and its generalization (KT solution) and argued that these admit descriptions in terms of the dS/CFT correspondence. Working on the gravity side we tried to get some insight on the properties of the *putative* dual CFT. The powerful tool that we used to do this is the counterterm prescription [18] for the renormalization of the field theory stress-tensor.

The AdS/CFT correspondence is a concrete realization of the holographic principle. Such correspondence is referred to as duality in the sense that the supergravity (closed string) description of D-branes and the field theory (open string) description are different formulations of the same physics. This way, the infrared (IR) divergences of quantum gravity in the bulk are equivalent to ultraviolet (UV) divergences of dual field theory living on the boundary. When we specify the CFT and say on which space it lives we are implicitly providing a set of counterterms for the gravity solution. These counterterms are local and depend only on the intrinsic boundary geometry [5].

In ref. [15], Strominger proposed a dS/CFT duality: bulk dS physics is dual to a boundary CFT (consequently, bulk quantum states are dual to CFT states on the boundary) and bulk time translation is dual to the boundary scale transformation. Since the bulk isometry group SO$(D,1)$ agrees with the boundary conformal group of a single euclidean CFT in D dimensions, it seems the dS/CFT correspondence involves a single dual field theory[5] (see, e.g., ref. [61] for a nice discussion). In the counterterm prescription we should also specify the surfaces on which the counterterms have to be integrated (equivalently, we should choose the space over which the dual field theory is defined). Note that these surfaces do not in general enclose anything — instead the conserved quantity is associated with the boundary surface itself. This situation is analogous to that in asymptotically AdS

[5]ref. [62] it was suggested that the duality should involve two CFTs and dS spacetime is defined as a correlated state in Hilbert space of the two field theories.

JHEP01(2004)029

or flat spacetimes; although the boundary surfaces in these cases enclose a bulk spatial region, the conserved quantities are completely insensitive to what transpires in the bulk, provided no information travels from bulk to boundary (or vice-versa) [4].

We note that in general the counterterm method does not always work. For example, in dS spacetime in global coordinates, the action is finite in even spatial dimensions only up to a term that linearly diverges in the cosmological time $\mathcal{T}$. Since all such boundary counterterm invariants are independent of this quantity, they cannot render the action finite [19]. This resembles the AdS/CFT case with two disconnected boundaries. However, in dS case the boundaries $I^{+/-}$ are causally connected and are two different slices of the same foliation.

Banks conjectured in ref. [17] that a quantum theory of gravity wih $\Lambda > 0$ has a finite number of degrees of freedom. The holographic screen is at the cosmological horizon, that is the largest surface from which an inertial observer can receive information. This proposal led to a conjecture by Bousso [40], referred to as the N-bound: "any asymptotically dS spacetime will have an entropy no greater than the entropy π/H^2 of pure dS with cosmological constant $\Lambda = 3H^2$ in $(3+1)$ dimensions". From this the authors of ref. [5] were further led to the conjecture that "any asymptotically dS spacetime with mass greater than dS has a cosmological singularity" (see, e.g., ref. [63]). Roughly speaking this latter conjecture implies that the conserved mass of any physically reasonable asymptotically dS spacetime must be negative (i.e. less than the zero value of pure dS spacetime). It has been shown that both of these conjectures can be violated for asymptotically dS spacetimes with non-zero NUT charge [64] (see, also, ref. [65]). However we have found that asymptotically dS spacetimes with a U(1) charge respect both conjectures.

Many properties of RNdS black holes that are obtained from a static coordinate system remain valid in a planar coordinate system that makes evident the expansion of the universe. The counterterm prescription yields a vanishing Casimir energy and the right expression for the gravitational mass, while the entropy of the cosmological event horizon can be written in a Cardy-Velinde form. The Cardy-Verlinde formula has recently been shown to be applicable for the entropy of topological black holes [66]. Althoug this formula was verified in the AdS/CFT context also, its interpretation is unclear. The main reason is that modular invariance, which is at the base of proving the Cardy formula, is a characteristic feature of two-dimensional CFTs only.

If there is a dS/CFT correspondence one is tempted to interpret time translation in the bulk as a boundary scale transformation [50]. By generalizing this beyond pure dS, time can be holographically reconstructed: time evolution in an asymptotically *expanding* dS space is equivalent to an *inverse* RG flow. Using different foliations for the RNdS spacetime, we obtain a time-dependent effective cosmological constant associated with the slices of the foliation. In this way, we provide nontrivial examples of the generalized dS c-theorem [51]: "the effective cosmological constant is larger in the interior of the space than at the (conformal) boundary". The interpretation of this result is less clear than in AdS/CFT case (see, e.g, ref. [51] for a discussion on "accessible" and "available" degrees of freedom on a given time slice). In our case, it would be more natural to interpret the inverse RG flows from a state with less entropy to a

JHEP01(2004)029

pure dS state. That is, the black hole is evaporating and the area of the cosmological horizon is increasing, approaching the area of a cosmological horizon of a pure dS spacetime.

The planar coordinate system admits also multi-black hole solutions with possible relevance for the cosmic censorship. An analysis of these configurations similar to the single-black hole case seems difficult. The boundary stress tensor presents divergences, which can be interpreted as reflecting the existence of the locations of the black hole sources in the bulk theory.

The KT multi-black hole solution (3.1) has been the starting point in a number of studies of cosmic censorship [45]. It would be interesting to understand this better from the point of view of the holographic theory. However, it seems to be rather difficult to propose a consistent CFT description of this type of dS solutions. For example, let us consider the case of two "ingoing" KT black holes with mass M_1 and M_2. At sufficiently early times, the holes are far apart (out of causal contact) and near each one the metric approaches that of a single black hole. Thus we have a state of approximate thermal equilibrium for every black hole with its own dS horizon, at temperatures T_1 and T_2. This is a consequence of the existence of approximate symmetries of the form (2.28) in the region near each black hole. Later the black holes come into causal contact and merge into a single black hole. The dS and black hole horizons approach a state of thermal equilibrium at a common temperature $T(M_1 + M_2, \Lambda)$. The topology of the boundary is expected to change when the two black holes merge.

Here, the existence of an upper limit (Nariai mass) for the total mass implies some strange results. As discussed in ref. [45], for $N = 2$ colliding black holes that are each less than the extremal mass but whose sum is greater, do produce naked singularities (we expect this result to hold also for $N > 2$). One can ask whether we can find a well-defined euclidean CFT dual to this situation. Late times in the bulk are supposed to correspond to the UV of the boundary theory, while early times correspond to the IR. Therefore we expect the dual theory to have highly non-equilibrated IR states, whereas the naked singularity in the bulk will manifest itself in UV pathologies of the CFT.

Let us take a closer look at this situation. There are some subtleties to deal with before one can conclude that cosmic censorship is violated in the KT solution. The naked singularity exists even before the black holes collide, due to the singular initial conditions we choose. This problem can be avoided by introducing a charged shell of dust that hides the naked singularity. On the other hand there is always a generically singular Cauchy horizon that has to be traversed by obsevers to see the naked singularity. The black hole collision is only possible in the contracting patch ($H < 0$), since the KT solution describes masses at arbitrary positions but not arbitrary velocities. If two undermassive black holes merge into an overmassive one, an observer at rest in contracting patch has no way of telling what is happening in the other "half" ($H > 0$) of dS spacetime. She only can guess the total mass will be $-(M_1 + M_2)$ to balance the mass she sees. Crossing the cosmological horizon ($r = +\infty$), the masses will appear to her very close together corresponding to very early time. However it is reasonable to interpret the cosmological horizon discontinuities as pulses of gravitational and electromagnetic radiation. It is clear now that for generic boundaries conditions there will be gravitational waves traveling along the Cauchy horizon.[6] When we give a particular spacetime which is asymptotically dS, we are giving a dual CFT living on the boundary and a particular state in the CFT. Matching smoothly expanding and contracting patches across the cosmological horizon implies a very precise selection for the CFT conditions (i.e. very special geometric data on the boundaries). It was shown in ref. [45] (see, also, ref. [46]) that this happens in a very special situations: several KT masses symmetrically distributed about a given one. If the observer survives crossing the Cauchy horizon, then cosmic censorship is violated in these examples.[7]

We ended with several remarks on a possible way to generate new solutions with a negative cosmological constant starting with EM-dS configurations in planar coordinates. We found that double analytic continuations of coordinates of the class of metrics in planar coordinates yield a class of bubble-like metrics given by eq. (5.2). These metrics are not bubbles in the sense that there is no additional periodic coordinate outside of the de Sitter subspace. However such metrics in principle furnish time-dependent backgrounds for string theory — their ultimate utility remains to be seen.

As a final comment we note that the counterterm method utilized here is a perfectly valid tool for the regularization of infrared divergences and the calculation of the gravitational mass of asymptotically dS spacetimes even when removed from the context of the dS/CFT correspondence. Therefore, while we are optimistic regarding this correspondence (however, see ref. [67]), the results described here hold independently of its validity.

Acknowledgments

D.A. would like to thank Robert Myers for bringing the refs. [27, 45] to his attention, at the beginning of this work. The authors would like to thank Robert Myers for valuable discussions and David Winters for proof-reading an earlier draft of this paper and for help with the figures. D.A. would like to thank the University of Waterloo's Department of Physics, ICTP Trieste, UBC's Department of Physics and PIMS Vancouver for their hospitality during various stages of this project.

The work of D.A. and R.M. was supported in part by the Natural Sciences and Engineering Research Council of Canada. D.A. is further supported by a Dow Hickson Fellowship. The work of E.R. was performed in the context of the Graduiertenkolleg of the Deutsche Forschungsgemeinschaft (DFG): Nichtlineare Differentialgleichungen: Modellierung, Theorie, Numerik, Visualisierung.

[6] We thank Robert Myers for discussion on this point.

[7] However, this is still not considered a serious violation of cosmic censorship. It was argued in refs. [45, 46] that if we want to consider a more physical situation (to form black holes from collapsing dust), the dust ball to form the black hole and the dust ball that hides the "overmassive" singularity collide before any singularity can form. Unlike the collision of two eternal black holes (for which the initial data contains the infinite throats of the black holes), it is not clear that starting with more generic initial data (compact data) the singularity at the Cauchy horizon is not stronger than in the KT solution.

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